Equilibrium in Wholesale Electricity Markets

Nathan Larson and David Salant*

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Abstract

We develop game theoretic models to evaluate strategic behavior in deregulated electricity markets, with particular attention given to the market rules in place in California through the summer of 2000. We prove existence of a Nash equilibrium under two particular sets of market rules used by the CALPX and CAISO respectively. Next we derive a lower bound (strictly above marginal cost) on average equilibrium prices when there is a positive probability that at least one generator is capacity-constrained. Finally, we compare two competing methods for modelling competition in power markets: supply function equilibrium and discrete, multi-unit auctions and illustrate shortcomings of both approaches.

1 Introduction

In this paper, we develop a series of conceptual models in an effort to identify the underlying factors behind the transaction prices and quantities in the California day-ahead, hour-ahead and real-time energy markets during the so-called "crisis" times in 2000 and 2001. Specifically, we show that, under conditions likely to have obtained in a significant number of hours during 2000 and 2001, prices were likely to have risen to levels above marginal costs purely as the result of the trading rules. The trading rules of the California Power Exchanges (CALPX) and the California Independent System Operator (CAISO) practically guaranteed that prices would rise above costs under competitive conditions, when individual generators were competing to maximize profits, and absent any implicit or explicit collusion.

The precise trading rules and the supply and demand factors affecting the California and related energy markets during the crisis period are extremely complicated and analysis of these markets to date has generally abstracted from many of these specific details. In the first part of the paper, we attempt to specify relatively precisely the game played by generators in each of the California power markets and prove the existence of a Nash equilibrium for each. Because of the discontinuities that the trading rules introduce in generators' payoffs, this is not a trivial exercise. In the second part of the paper, we illustrate through a series of simple models that when the supply of every generator may be needed to meet peak demand, equilibrium prices will tend to exceed marginal cost. Under fairly general conditions on demand, we derive a lower bound on the difference between average equilibrium prices and marginal cost. Here, as elsewhere in the paper, we analyze market outcomes in the absence of tacit collusion, so

*Affiliations and contact information: Nathan Larson, University College London and National Economic Research Associates, n.larson@ucl.ac.uk. David Salant, National Economic Research Associates, David.Salant@nera.com
this result casts doubt on assertions that market prices in California that were substantially above marginal cost could only be the result of collusion. In the final section of the paper, we discuss the conflict between two ways to model equilibrium in power markets: the supply function equilibrium approach of Klemperer and Meyer and the multi-unit auction approach of von der Fehr and Harbord. Under the former approach, generators are assumed to offer smooth, continuous supply schedules to the market. Although in practice, market rules and generation technology are such that supply schedules must be somewhat “lumpy”, the SFE approach assumes that the approximation error thus introduced is minor. The latter approach assumes that discreteness in supply schedules cannot be safely ignored, and assumes that generators offer supply to the market on a unit-by-unit basis. We investigate the tension between these approaches in a succession of simple models and demonstrate that both approaches have shortcomings. In particular, the assumption that supply schedules can be safely approximated by continuous curves is shown to be unjustified when schedules must take the form of step functions. However, we also show that although the multi-unit auction approach can sometimes predict equilibria with prices at or near marginal cost, these equilibria often are not stable when generators update their bids according to simple learning rules. As the high dimensionality of the strategy space when supply schedules are taken as discrete makes analytical results hard to come by, we illustrate our results principally through simulations.

In the next section, we provide a brief description of the market rules in effect in California during the summer of 2000. Section 3 presents our analysis of the existence of Nash equilibrium in these markets. Section 4 presents our lower bound on prices. Section 5 presents our simulations comparing the supply function and multi-unit auction approaches, and Section 6 concludes.

2 Overview of the California Power Markets

In the California energy markets and the plural is important - there were multiple opportunities for trade, a supplier’s decision about which market or markets to bid in and how was a strategic choice that would affect the outcome. While the CALPX was in operation, the California energy market included a block-forward market, a day-ahead market for hourly contracts, and a post-close quantity match all administered by the CALPX. Subsequent to trade in the CALPX day-ahead market, the CAISO administered an hour-ahead market. These markets had different bidding rules, and energy not purchased or sold in the block forward or day-ahead CALPX markets could be purchased or sold in CAISO markets. There were numerous other trading opportunities for the sellers, and some buyers. Regulation required some buyers to make all their purchases through the CALPX and CAISO. Some of the buyers were net buyers, in that they also had their own energy resources. In summary, the trading environment was quite complex. This section describes the rules that applied to the main constituent markets within this trading environment, and asks whether these individual markets, treated in isolation, have Nash equilibria. Answering this question is important if we want to know how confidently we can bring the standard tools of economic theory to bear on behavior in the California power markets. Of course if the rules of the some of the power markets in California do not admit equilibrium, this does not mean that economic analysis of those markets is bankrupt. but it does mean that additional assumptions must be made, and defended, in regard to how market
participants are likely to behave in a disequilibrium situation. Before proceeding, we also note that although demonstrating the existence of equilibrium in individual markets in isolation is an important first step, this by no means guarantees that an equilibrium will exist in the larger game formed by interlinking these constituent markets.

To examine the impact of multiple trading opportunities that occur in sequence, the appropriate approach is to solve the game backwards. In particular, this means that the analysis of equilibrium in one of a sequence of opportunities to sell in wholesale energy markets, must take into account the impact of the trades in that market have on what happens subsequently in the other subsequent markets for trades.

To determine existence and nature of equilibrium for the entire game, we therefore need to start with the last trading opportunity. In the case of the California energy markets from 1998 – June 2001, this analysis would start with the CAISO hour-ahead market. When a bidder reaches the last opportunity for trade, that bidder will want to submit a bid strategy to maximize expected payoffs, given what that bidder’s beliefs about likely demand and the strategy of others supplying into that market. So, to analyze what constitutes an equilibrium strategy, or even if equilibrium exists in the hour-ahead market, one can take as given the results of the CALPX day-ahead market and the outside options. In what follows, we describe some possible features of a market, such as the CAISO hour-ahead market, in which bidders are required to submit step function bids.

2.1 The CAISO Hour-Ahead Market

The CAISO hour-ahead market was principally intended for reserve energy needed to meet real-time imbalances, with the bulk of capacity and scheduled load being into one of the earlier CALPX markets, although in practice this was not always the case. The CAISO operated a number of sub-markets, which were essentially cleared sequentially. Some types of reserves have shorter response times than others, and the last to clear were those having the shorter response times – in order of increasing response times – these resources are regulation, spin, non-spin and replacement reserves. Bids by generators to supply capacity for use as system reserves had to specify the generating unit being offered, as differences in response times or location could make it necessary for the system operator to upon specific units. A bid consisted of a single price, representing a capacity payment to be made to the generator regardless of whether the capacity it offered was needed, and a supply schedule specifying the amount of power it would be willing to have dispatched from that unit at a sequence of prices. In our current analysis, we will leave aside the issue of the capacity payment and focus on the bidding in supply schedules.

The CAISO required supply schedules bid to be step functions with up to ten steps. This means that a bid could be characterized as a sequence of ten pairs \((p_f, q_f)\) in \(20\), where \(p_f = p_{f+1}\) and \(q_f = q_{f+1}\) for all \(f = 1, 2, \ldots, 10\). The interpretation is that \(p_1\) is the price the firm would want for the first \(q_1\) MWs, \(p_2\) the price for the next \(q_2\) MWs, etc. In what follows, we consider the case in which there are two firms. The firms have a finite number of units. Each unit \(u\) of firm \(f\) has a finite capacity \(K_{fu}\) and constant unit costs up to capacity of \(c_{fu}\), and \(c_{fu} = c_{fu+1}\) for all \(u\) and \(f\).

Demand is then allocated to the firms offering the lowest prices. The tie-breaking rules for allocating demand can matter. In what follows, we assume that a random selection is made when two or more firms submit equal prices for one step, and that the entire supply offered
by the first firm chosen at that price will be taken prior to the next firm offering an amount to supply at that price. This is essentially equivalent to using a time-stamp, as is common practice, for breaking ties.

One simple and immediate observation about this auction is that the quantity allocated to any one bidder is not a continuous function of its bid. This means that payoff functions are not continuous either. In particular, assuming that price can vary continuously, each firm will want to undercut its rival by an infinitesimal amount, at least for its last step in its step function offering.

These undercutting incentives tend to rule out pure-strategy equilibria in many cases. This means that equilibrium, if it exists, tends to require each firm to randomize, that is, avoiding being too predictable. Predictable bidding is costly in that it results in being undercut. When prices can assume only a finite number of values, and the classic Nash result would apply. When prices can vary in very small amounts, then the above suggests that if equilibrium exists it will have to be in mixed strategies in the finite game as well. The question is then of whether the restriction to a particular finite set of prices matters. In particular, should equilibrium fail to exist when price can vary continuously, it can be the case that the particular finite set of prices can affect the nature of equilibrium distribution. Indeed, when payoff function discontinuities are too irregular in a sense defined more precisely below (and by Dasgupta and Maskin (1986)), then the limit of finite games will not necessarily converge. What this suggests, and what our subsequent simulations support, is that equilibrium in the finite strategy-set games will be very unstable, or very sensitive to parameter values.

One form or continuity which ensures convergence of the equilibria of a sequence of finite games to equilibrium of the continuous game is upper-semicontinuity. Unfortunately, it turns out that these payoffs need not be upper nor lower semi-continuous. But, for at least the two-player symmetric game, we can appeal to a result of Dasgupta and Maskin to show existence when the strategy set is limited to setting one price for the entire capacity. What is required is that in some sense the game is that at any jump discontinuities, the two players’ payoffs jump in opposite directions. Note, that strategies can be characterized as prices, \( p_j, j = a, b \), with \( 0 \leq p_j \leq p^c \), where \( p^c \) is the maximum price for which demand is non-negative. In what follows, we assume that demand is allocated evenly when two firms quote the same price for one step, and that overall demand is continuous. We let \( \Pi_j(p_A, p_B) \) denote the profit of firm \( j \) at \( (p_A, p_B) \). We assume, for sake of simplicity that there are no costs. In what follows, we can show that an equilibrium exists in the two person game when \( N = 1 \) and the two firms have equal capacities.

**Proposition 1** The two-player step function auction has an equilibrium in mixed strategies when both firms have the same capacities.

**Sketch of proof:**

The profits are continuous except where \( p_i = p_j \) for some two prices offered by the two different firms.

To show existence, it suffices, by Theorem 5b of Dasgupta, Maskin (1986a), to show that for each \( p \in [0, P] \),

\[
(i) \lim_{p_A \to p^-, p_B \to p^+} \Pi_i(p_A, p_B) \geq \Pi_i(p, p) \geq \lim_{p_A \to p^+, p_B \to p^-} \Pi(p_A, p_B) \text{ and }
\]
\[(ii) \lim_{p_A \to p^{-}, p_B \to p^{+}} \Pi_j(p_A, p_B) \leq \Pi_i(p, p) \leq \lim_{p_A \to p^{+}, p_B \to p^{-}} \Pi_j(p_A, p_B), \quad \text{ji.}\]

where the left (right) inequality is strict in the first expression if and only if the right (left) inequality is strict in the second expression.

The discontinuities in profits occur when \(p_A = p_B = p\). At such a point, if \(p_A\) approaches \(p\) from below, then (ii) is satisfied for firm A and (i) is satisfied for firm B.

Therefore, by Theorem 5b of Dasgupta-Maskin (1986a), the game has an equilibrium in mixed strategies.

Note that in an N-person auction, one firm’s gains are not solely at the expense of one rival. Moreover, the sum of the payoff functions need not be upper semi-continuous. For this reason, Theorem 5 and its corollaries in Dasgupta and Maskin (1986a) do not apply directly. Dasgupta and Maskin (1986b) considered another game, of insurance contract competition, which failed to satisfy the upper-semicontinuity of the sum of payoffs as well. The payoff functions here, as in the model of Section 4 of Dasgupta-Maskin (1986b), are continuous except on a set of measure 0 that satisfy (A1) of Dasgupta-Maskin (1986a), i.e., \(U_j = U_k\) for some \(j \neq k\). As in Theorem 5 of Dasgupta-Maskin (1986b), we can define modified payoffs which have an upper-semicontinuous sum, and also satisfy a version of weak lower semi-continuity, labeled Property \(\alpha^*\) in Dasgupta-Maskin (1986a). Given this is the case, then an M firm game with N steps will have an equilibrium in mixed strategies, with the modified payoffs. Property \(\alpha^*\) essentially ensures an atomless equilibrium distribution for the modified game. And if the distribution is atomless, the same strategies will be an equilibrium for the original game, as the payoffs can only differ on sets of measure zero.

2.2 The CALPX “Day-Ahead” Market

The bulk of supply and demand on a daily basis was intended to be scheduled in one of two markets run by the CALPX, the day-ahead and day-of markets. The two markets had similar bidding rules, although in practice, there was virtually no trading volume in the day-of market. So, we focus on bidding in the day-ahead market. In this market, generators submitted independent supply schedules for each hour of the following day. Each supply schedule was permitted to consist of up to sixteen price-quantity pairs. Successive pairs were connected by straight lines to form a continuous, piece-wise linear supply curve with up to fifteen line segments. Furthermore, these supply curves were required to be strictly upward sloping – in contrast with the requirement of step function bids in the CAISO hour-ahead market. The market was cleared by aggregating these supply curves across all bidders and equating supply with demand. The framework also included the possibility of decremental bids that could be called upon to reduce excess generation and mitigate congestion; we will abstract from this.

It can be seen that the profit functions, even with zero costs, with have jump discontinuities at prices at which two or more firms have supply schedules with initial jump discontinuities, that is, supply jumps from 0 to some positive finite amount a minimum price. Otherwise, the individual firm payoffs will be continuous. With piecewise linear supply, firm supply is of the form \(q = a + bp\) at each point, and the entire supply for a firm is a set of values for \((a_j, b_j)\) and ranges over which they hold \(p_0, p_1, \ldots, p_n\), with the supply being \(q_j = a_j + b_j p\) for \(p_{j-1} \leq p \leq p_j\). Further assuming no initial jump discontinuity, or flat segment in supply, and that otherwise \(b > 0\) for all firms, then the demand function facing each firm will be continuous as a function.
of \( a_j, b_j \) and \( q_j \). In this case, the payoff functions are continuous over a compact set, and the game then has an equilibrium in mixed strategies (see Glicksburg (1952)).

3 Derivation of a Lower Bound on Prices

The purpose of this section is to derive a lower bound on the average market-clearing price that can be expected when two suppliers, each facing a capacity constraint, compete to meet an uncertain demand by submitting supply schedules. For simplicity, the marginal cost of each supplier is normalized to zero, and the maximum capacity that each supplier can provide is assumed to be \( K \). There is assumed to be a maximum clearing price \( p^c \), and we denote by \( Q^c \) the quantity demanded at this "cap" price. This quantity is uncertain from the point of view of the suppliers.

Although the derivation of the bound requires several steps, the logic of why there must be a lower bound on prices is straightforward. Suppose that the suppliers knew for sure that demand at the cap price would exceed the amount that either supplier could provide independently by at least \( d \): \( Q^c > K + d \). Then by offering to supply only at the cap price, a supplier would be assured of providing at least \( d \) units at a price \( p^c \), earning a profit of \( p^c d \). Alternatively, it could offer a supply schedule including a positive quantity offer \( q \) at some price \( p \) below \( p^c \). In the event that this price clears the market, the bidder must earn a profit greater than \( p^c d \) - otherwise it would be better simply to only offer at the cap price. The profit earned is \( pq \), which must be less than \( pK \) (because it cannot supply more than \( K \)). But this means that prices \( p < (d/K)p^c \) will never be offered, because for these prices \( pq < pK < p^c d \). Thus, the certainty of earning at least \( p^c d \) by serving overflow demand makes suppliers unwilling to offer very low prices.

The complexity arises because demand is uncertain, so the minimum profit earned by charging \( p^c \) is not certain as assumed above. When deciding what price to charge for small quantities, a bidder must consider that it will tend to sell these quantities when demand is relatively weak, in which case the alternative (charge \( p^c \) and serve the overflow) is relatively unattractive. This means that the lower bound on the price for small quantities can be quite low. On the other hand, the price for large quantities must be relatively high, because large quantities sell when demand is strong - which is precisely when the option of charging \( p^c \) is most attractive. For this reason, it is sensible to look for a bound on average market-clearing prices relative to the average strength of demand.

The strategy for deriving the lower bound resembles the argument sketched above. First, we show that the expected profit earned by a supplier who charges \( p^c \) for all quantities between 0 and \( K \) is at least equal to \( \pi^c \) (for some \( \pi^c \) to be derived). Then we show that there is a relationship between the average market-clearing price and the total expected profits of both suppliers: if the average price is \( \mu \), then total expected profits must be less than \( \pi^{ub}(\mu) \) (where "ub" signifies "upper bound"). Finally, we argue that if total expected profits are not at least \( 2\pi^c \), then one of the two suppliers could do better by always charging \( p^c \). This allows us to put a lower bound on \( \mu \).

Step 1: Deriving \( \pi^c \)

6
Suppose that Supplier 1 submits the following supply curve:

\[
\text{Quantity supplied} = \begin{cases} 
0 & \text{if price } < p^c \\
\text{any quantity between 0 and } K & \text{if price } = p^c
\end{cases}
\]

Then whenever demand at \( p^c \) exceeds the capacity constraint of Supplier 2, Supplier 1 will sell at least \( \min\{Q^c - K, K\} \) units at price \( p^c \). (That is, it will serve all of the overflow, unless the overflow is larger than its own capacity, in which case it will supply \( K \).) Thus, its profit for a particular realization of demand is at least \( p^c \min\{Q^c - K, K\} \) if \( Q^c > K \) and at least 0 otherwise. Its expected profit is then at least

\[
\pi^c = p^c \Pr(Q^c > K) E[\min\{Q^c - K, K\} | Q^c > K]
\]

where \( \Pr(Q^c > K) \) is the probability that \( Q^c \) exceeds \( K \), and \( E[\min\{Q^c - K, K\} | Q^c > K] \) is the expected quantity sold by Supplier 1 in the event that \( Q^c \) exceeds \( K \).

**Step 2: Deriving \( \pi^{ub}(\mu) \)**

Now suppose that the two suppliers play a Nash equilibrium in which they earn expected profits \( \pi_1 \) and \( \pi_2 \) respectively. The market-clearing price in this equilibrium will vary randomly due to the uncertainty in demand (and also because, in some cases, it may be optimal for suppliers to randomize their bids). For any realization in which the market-clearing price is \( p \), the total profit earned by both suppliers must be less than \( 2pK \) (because between them, they cannot supply more than a quantity \( 2K \)). But then, their total expected profits \( \pi_1 + \pi_2 \) over all possible levels of the market-clearing price must be less than

\[
E[2pK] = 2K E[p] = 2K \mu
\]

Thus, given the average market-clearing price \( \mu \), we can place an upper bound \( \pi^{ub}(\mu) = 2K \mu \) on the sum \( \pi_1 + \pi_2 \).

**Step 3: Deriving the lower bound on \( \mu \)**

In any Nash equilibrium, the expected profit earned by each supplier under its equilibrium strategy must be greater than the profit it could earn by always charging \( p^c \). Thus,

\[
\pi_1 \geq \text{[Profit from "always \( p^c \)]} \geq \pi^c
\]

and

\[
\pi_2 \geq \text{[Profit from "always \( p^c \)]} \geq \pi^c
\]

This means that the sum \( \pi_1 + \pi_2 \) must be at least \( 2\pi^c \). Now using the upper bound derived in Step 2, we have

\[
\pi^{ub}(\mu) \geq \pi_1 + \pi_2 \geq 2\pi^c
\]

If this expression does not hold, then total profits cannot be large enough to prevent at least one of the suppliers from switching to "always \( p^c \)." Using the expressions derived for \( \pi^{ub}(\mu) \)
and \( \pi^c \), we can rewrite this expression as:

\[
\mu \geq p^c \Pr(Q^c > K) E[\min\{Q^c/K - 1, 1\} | Q^c > K]
\]

This lower bound on the average market-clearing price depends only on the average strength of demand relative to the capacity limits of the suppliers, as indicated by the terms within the brackets. When demand is stronger, the chance that one of the suppliers will be capacity-constrained is greater, and average prices must be bounded further from marginal cost. Furthermore, a positive lower bound exists whenever there is some chance of overflow demand at the cap price (\( \Pr(Q^c > K) > 0 \)), even when this chance is quite small.

**Example**

As an example, suppose that demand has the following form.

\[
\text{Quantity Demanded} = K + D \quad \text{for} \quad p \leq p^c
\]

\[
= 0 \quad \text{for} \quad p > p^c
\]

\( D \) is a random shock that is distributed uniformly on the interval \([-d, d]\). Thus, on average, demand is just equal to the amount that each bidder can supply independently, but on any given day it may be greater or less than this. We can apply the lower bound formula as follows. Demand at the cap price is just \( K + D \), so the probability of overflow demand is \( \Pr(Q^c > K) = \Pr(D > 0) = \frac{1}{2} \). To keep the calculation simple, assume that \( d < K \), so that demand at the cap price never exceeds the total capacity of both firms: \( Q^c < 2K \). (Also, this ensures that demand is always positive.) Then, in the event that demand exceeds \( K \), the average amount of the overflow \( Q^c - K \) is just \( d/2 \), so we have

\[
E[\min\{Q^c/K - 1, 1\} | Q^c > K] = E[Q^c/K - 1 | Q^c > K] = \frac{d}{2}K
\]

Therefore, the lower bound on the average market-clearing price is

\[
\mu \geq p^c \frac{dK}{4}
\]

When \( d = 0 \), so that demand is equal to \( K \) with certainty, the bound puts no constraint upon the clearing price. In fact, one equilibrium in this case (there are others) is for both suppliers to charge \( p = 0 \) for any quantity. If either supplier were to try to charge a higher price, the other supplier would serve the entire demand.

However, when \( d \) is positive, although either of the suppliers can independently serve the average level of demand, there are positive profits (\( dp^c/2 \) in expectation) to be earned in the event that demand is stronger than average. Given this chance, a rational supplier would never want to deny itself any possibility of earning appreciable profits by bidding a very low price. When \( d \) is larger, the upside profits from selling to overflow demand are larger as well, so average prices must rise. When \( d = K \), so that demand varies uniformly between 0 and \( 2K \), average prices must be at least 25% of the cap price. If we allow for the possibility that demand outstrips the capacity of both suppliers, the lower bound would rise even further.
By design, the construction of the bound makes almost no assumptions about the shape of demand. As a result, the upper bound on total profits $\pi_{ub}(\mu)$ is likely to substantially overestimate the expected quantity sold, given an average price of $\mu$. With more information about the shape of demand, this upper bound could be tightened, and a larger range of average prices could be ruled out. Extending this bound to handle the case in which marginal costs are increasing would also be relatively straightforward. In the appendix, we show that the same technique can be applied to derive price bounds with any number of firms with (possibly different) capacity constraints.

In principle, all of the information needed to compute a numerical value for the lower bound on prices is available. The distribution of demand can be reconstructed from historical data, and the operating limits of plants should be public. However, the issues of which plants are marginal at any point in time, and what bidders can be expected to know about the likelihood of other bidders' capacity constraints binding would probably need to be handled delicately.

4 Supply Function Equilibria vs. Multi-unit Auctions: How Much Does Discreteness Matter?

With the recent rapid pace of deregulation, markets for electricity generation are now commonplace. In a broad sense, many of these markets share a similar format in which individual generators simultaneously submit schedules specifying the amount of power that each is willing to supply at various prices for a particular time period. These “supply curves” are added to form an aggregate supply curve which is then crossed with aggregate demand to determine how much generation each supplier will dispatch, and at what price.

The devil, of course, is in the details. Implementing this market template requires more specific choices on a number of issues, including pricing (should it be uniform or pay-as-bid?), the time period over which bids are binding (can supply schedules be updated on an hour-by-hour basis, or must a generator commit to a single schedule for an entire day?), and the format in which supply schedules must be submitted, among others. Furthermore, modelling outcomes in such a market requires making reasonable assumptions about the elasticity and variability of demand, the shape of generators' marginal cost curves (and the importance of start-up costs), the information available to the bidders (and perhaps the likelihood of collusion), and so on. This paper will focus on one particular issue that has proved divisive for modellers: how (and whether) to account for discreteness in generators' supply schedules.

There are two principal reasons to think that a discrete approach to modelling supply schedules might be appropriate. The first is technological: each generator's capacity is divided among a finite number of plants, and the fixed cost associated with starting up a plant can be substantial relative to the marginal cost of supplying an additional unit of power once the plant is up and running. Thus, it may be natural for a generator to develop its pricing strategy on a plant-by-plant basis. The second reason has to do with the market rules: in practice, all operational markets restrict bidders to submitting a finite number of price-quantity points rather than a continuous curve. On this basis, von der Fehr and Harbord pioneered the approach of modelling competition among generators as a multi-unit auction in which each generator offers a single price for each of a finite number of generation units. In contrast, under the supply function equilibrium of Klemperer and Meyer, which was first applied to
power markets by Green and Newberry, generators are assumed to be capable of submitting continuously differentiable supply functions. Proponents of this approach argue that when the number of discrete steps available to bidders is relatively large, when lumpiness in bids is aggregated over a number of suppliers, and when there is some uncertainty about supply (due to unplanned outages, for example) and demand, then the expected residual demand faced by bidders will be relatively smooth, and their optimal supply schedules will be closely approximated by continuous functions.

The distinction between the two approaches is not just academic; they predict dramatically different market outcomes. In the multi-unit auction framework, pure strategy equilibria (when they exist) tend to exhibit Bertrand-like pricing - that is, prices at or near marginal cost. The driving force for this result is familiar from basic models of Bertrand competition: by undercutting a competitor’s price by an infinitesimal amount, a generator can win a non-infinitesimal increase in its quantity dispatched. In contrast, with continuous, upward-sloping supply curves, undercutting a competitor only allows a generator to capture an infinitesimal increase in quantity, so prices in a SFE are typically not driven down to marginal cost. In fact, because there is a sort of strategic complementarity among generators - the less elastic your supply curve is, the less elastic my residual demand curve will be, and hence, the steeper my optimal supply curve - SFE models generally admit a range of different equilibria, with peak prices ranging between the competitive and Cournot levels. Depending on parameters, the latter may entail markups of perhaps several times marginal cost. Clearly, if this sort of modelling is intended to eventually inform market design and competition policy, the question of how two reconcile these two approaches, or at least establish conditions under which one is more appropriate than the other, seems pressing.

In this section, we take a small step this direction by exploring the behavior of bidders who must submit step-function bids and who adapt their bids over time using simple learning rules. Our current results are quite limited and are chiefly confined to simulations of some stylized examples. Nonetheless, they are suggestive, as they indicate that under some circumstances neither the multi-unit auction nor the SFE framework will adequately predict market outcomes. For example, we will illustrate that in games with unique Bertrand-like pure strategy equilibria, bidders may nonetheless converge to an approximate mixed strategy equilibrium with prices above marginal cost but well below the prices in an analogous SFE game. Furthermore, simulations with ten-step supply functions indicate that at this level of coarseness, long run bidding behavior can be unstable, with substantial price swings, and need not settle down in the neighborhood of a SFE. Although the example is stylized, the degree of discreteness imposed is not too far off the mark - in California, for instance, the real-time imbalance energy market run by the ISO required bids in the form of step-functions with at most ten steps.

Our computational investigation of bidding by power generators contributes to a growing literature on computing equilibria in supply function games. As we mentioned above, Green and Newberry (1993) pioneered the approach of numerically computing supply function equilibria. Their approach is based on assuming continuity of the supply schedules and quadratic costs, and numerically solving the differential equations that characterize different equilibria. Their method is agnostic about which SFE, out of the continuum of possible equilibria, will actually be observed. More recently, there has been interest in explicitly simulating the process by which generators adjust their bids from one day to the next; the hope is that this will not only provide
a natural way to compute equilibria but will also shed light on which equilibria are likely to
arise. Day and Bunn (2001) is a notable contribution in this vein. Finally, of substantial
interest for our work is Baldick and Hogan (2001), who show that when generators’ supply bids
are subject to a certain type of noise (and with quadratic costs and continuous bid functions),
only the linear SFE is stable. Their simulations generally confirm that under these conditions,
bidding near the linear SFE is to be expected. Our results present a more nuanced view. We
show that when discreteness is an integral feature of supply bids – even with a relatively fine
grid – price instability and cycles are robust market outcomes.

4.1 A Simple Model of Bidding with Discrete Units

This section illustrates how generator behavior can diverge from the predictions of the MUA
and SFE models through a series of examples. We begin by setting up a very simple game with
multiple discrete generation units in which there is a unique pure strategy outcome in which
the price is equal to the marginal cost of the most efficient unit not to be dispatched. We then
demonstrate that this equilibrium will not be realized under a plausible strategy adjustment
process. We simulate the outcome of this adjustment process numerically and demonstrate
that it is an approximate mixed strategy equilibrium. Finally, we define an analogous game
in which the generators can submit continuous supply functions and show that the outcome of
our adjustment process does not approximate a supply function equilibrium of this game.

4.1.1 The Benchmark Game

There are two generators, A and B, each of which has two discrete, equally sized blocks of
capacity to offer to the market. For each firm, the marginal cost of power supplied by its first
unit is 0 (across its entire capacity), and the marginal cost of power supplied by its second unit
is 1. Demand is inelastic and certain: it is equal to 2 units up to a cap price of 3. The
firms compete by bidding a single price for power from each of their units; that is, each
firm submits a pair of positive real numbers \((p_1, p_2)\). The market is cleared by ranking all of the
bids in ascending order to form an aggregate supply curve and selecting the total quantity to
clear supply and demand, so as long as there are at least two units offered at a price of 3 or
below, the units with the lowest and second-lowest prices are dispatched. Each dispatched
unit is paid the system price which is equal to the bid of the marginal dispatched unit (i.e., the
second-lowest bid). Ties are broken by dispatching the lower cost unit first, or by lottery if
the tied units have the same cost.\(^1\)

**Proposition 2** In every pure strategy Nash equilibrium in weakly undominated strategies of
the game described above:

i) The equilibrium price is 1.

ii) Each generator dispatches one unit and earns a profit of 1.

iii) Each generator bids its second unit at its marginal cost of 1.

**Proof.** First, observe that bidding its second unit below cost is a weakly dominated strategy
for each firm. Next, suppose there were an equilibrium with a system price \(p\) strictly greater
than 1. Total profits in such an equilibrium must be less than 2\(p\), so at least one firm, say

\(^1\)Under other tie-breaking rules, an equilibrium may fail to exist.
A, must be earning less than or equal to $p$. But then A could offer both of its units at $p' = p - \varepsilon$, thereby earning $2p - 1 - 2\varepsilon > p$ for $\varepsilon$ small enough, so this cannot be an equilibrium. Alternatively, suppose there were an equilibrium with $p < 1$. Because we have ruled out bidding below cost, each firm must be selling its first unit in this equilibrium and earning $p$. But then either firm could raise the price of its first unit to $1 - \varepsilon > p$ and continue to sell one unit but at a higher price, so this cannot be an equilibrium either. Therefore, the price in equilibrium must be equal to 1, so each generator earns profits of at most 1. Now suppose that some generator were to bid its second unit at a price $p_2 < 1$. Because we have ruled out bidding below cost, each firm must be selling its first unit in this equilibrium and earning $p$. But then either firm could raise the price of its first unit to $1 - \varepsilon > p$ and continue to sell one unit but at a higher price, so this cannot be an equilibrium either. Therefore, the price in equilibrium must be equal to 1, so each generator earns profits of at most 1. Now suppose that some generator were to bid its second unit at a price $p_2 > 1$. Then the other could boost the system price to $p_2 - \varepsilon$ by raising the price of its first unit, thus earning a profit greater than 1. Thus, each generator must bid its second unit at a price of 1.

Finally, note that bidding both units at a price of 1 is a symmetric equilibrium. Neither firm can do better by lowering the price of its first unit or raising the price of its second unit, as neither of these actions changes the equilibrium price or allocation. If a firm were to raise its first unit price, it would fail to sell, losing a potential profit of 1. Conversely, if a firm were to lower its second unit price, it would be forced to operate that unit at a loss. There are also asymmetric equilibria in which one firm bids $(p_1, 1)$ and the other bids $(1, 1)$, with $p_1 < 1$.

In passing, we also note that there is also a continuum of equilibria in weakly dominated strategies in which both firms bid both units at $p < 1$ for any $p \in [0, 1)$. In these equilibria, each firm is indifferent to bidding its second unit below cost because it knows that unit will never be dispatched.

This example exhibits the standard intuition associated with Bertrand-like pricing models: the system price is bid down to the marginal cost of the most efficient losing unit. If the price were any higher, the owner of this most efficient losing unit would have an incentive to undercut slightly in order to get this unit dispatched at a profit. Notice, as well, that in equilibrium the price at which a firm offers its less efficient unit is irrelevant for its own profits. One might try to motivate the decision to price this unit at marginal cost with a story along the following lines:

1. Suppose A were to set $p_2^A > 1$. Then,
2. B would respond by raising the price on its first unit to just below $p_2^A$, driving up the system price.
3. But then A would have an incentive to reduce $p_2^A$ to undercut B’s response, thereby dispatching his second unit at a profit.
4. But then B would respond by undercutting A again, and so on.

Presumably this process could end with prices converging back to 1. However, there is another possibility. At Stage 2, B must raise the price on its second unit by more than it raises the price of its first unit. (Otherwise it would end up dispatching the more expensive unit but not the less expensive one.) But B is more or less indifferent to how much more it raises the second unit price, since this unit is not expected to be dispatched in any case. (After all, A’s first unit is still priced at 1.) If B raises its second unit price enough, then at Stage 3, A may find it more profitable to respond by raising $p_1^A$ to just below $p_2^B$, thus selling only one unit, but at a high price, rather than reducing $p_2^A$ in order to sell two units at a lower price.
In this second story, the eventual convergence of all prices back to 1 seems less obvious, and the prospect of cycling or perhaps convergence to a mixed strategy equilibrium seems like a possibility. In the next section, we develop a model of a dynamic price adjustment process to replace the informal stories described above.

4.1.2 The Dynamics of Price Adjustment

Our price adjustment model is a version of stochastic best reply dynamics, and is based loosely on the idea that each generator tends to shift its pricing strategy toward bids that would be good responses to the historical pattern of prices chosen by its opponent. Let $f_i^t$ be the probability density function from which generator $i$ chooses its bid at time $t$. We assume that $f_i^0 > 0$ on $\{(p_1, p_2) : 0 \leq p_1 \leq p_2 \leq 3\}$, so that initially every feasible price pair has some possibility of being selected. Let $\pi(p, f)$ be the expected profit earned by bidding the price vector $p = (p_1, p_2)$ against an opponent playing according to the distribution $f$. We assume that the generators adjust their bidding according to the following system of equations:

$$f_i^t = (1 - \alpha)f_i^{t-1} + \alpha g_i^t \quad i = A, B$$  \hspace{1cm} (1)

That is, each generator’s distribution over bids is a weighted average of its distribution last period and an update term $g_i^t$. This update term places greater weight on bids that would have earned relatively higher profits against the distribution used by the opponent in the last period:

$$g_i^t(p) = \frac{e^{\kappa\pi(p, f_i^{t-1})}}{D_i^t}$$ \hspace{1cm} (2)

where $D_i^t = \int e^{\kappa\pi(p', f_i^{t-1})} dp'$ is just a normalization constant.

One can motivate these equations of motion as follows. After every period, a generator reviews the distribution of bids by its opponent in that period. (Think of a period as comprising a number of opportunities to bid - hourly markets within a day, for example - and that generators reassess their own strategies at the end of the day, after observing their opponent’s bids over the course of the day.) The generator attempts to calculate a best response to that opponent distribution, but does not do so perfectly - the update distribution $g_i^t$ it comes up with places greater, but not infinite, weight on more profitable responses. This could be because analyzing the market is costly and time-consuming or perhaps because of small unmodelled cost shocks that shift the optimal response. Finally, there is inertia in the generator’s pricing strategy: it merges its update over its optimal strategies with the distribution it used last period to determine its probability of making various bids today.

This adjustment process is more flexible than it may at first appear. Notice that equation (2) is just the familiar logit equation from discrete choice modelling. By setting $\kappa$ very high, one can model generators whose updates are arbitrarily close to placing full weight only on optimal responses. Furthermore, by additionally setting $\alpha$ to one, one can model generators as responding instantly and myopically to the last action chosen by their opponents as in the motivational stories from the previous section. On the other hand, if one believes that generators react gradually to changes in strategy by their opponents, then this can be incorporated by setting $\alpha$ lower.

Speaking loosely, we will call a pair of strategy distributions stable if the adjustment process converges to them from an open set of initial conditions. We have not tried to prove whether
(1) converges, although our simulations will shed some light on this. However, it is clear that if stable strategy distributions exist, they must satisfy
\[ f_i^t = e^{\kappa (p_i f_{-i}^{t-1}) / D_i^t} \]  
Equation (3) is a continuous version of the logit equilibrium defined by McKelvey and Palfrey (1995) for discrete strategy spaces. They show that solutions to (3) converge to Nash equilibria in the limit as \( \kappa \) grows large, so one can view the adjustment process as a way to predict which (approximate) Nash equilibrium is likely to be played. We will call a Nash equilibrium stable if it is the limit of a sequence of stable pairs of strategy distributions as \( \kappa \) goes to infinity. Our first result is the following.

**Proposition 3** The game has no stable pure strategy equilibria.

**Sketch of Proof**

Suppose there were a sequence of logit equilibria converging to the symmetric equilibrium in which both generators bid \((1,1)\). In order for second unit bids to converge to 1, it must be true that second unit bids near 1 sometimes win (at each of these logit equilibria), making it strictly more profitable to bid near 1 than at higher prices. But this means that first unit bids must sometimes lose, so first unit bids cannot converge to 1 at a faster rate than second unit bids. However, the rate at which each unit’s bid converges to 1 is driven by the loss in profits that would be incurred if that unit were bid too high. This loss in profits is roughly constant at 1 for the first unit but goes to 0 for the second unit, so the distribution of first unit bids must converge at a faster rate, contradicting the previous claim.

The driving force behind this result is the assumption that a generator will choose a bid relatively arbitrarily when it is perceived to be irrelevant to its payoff. Later we will investigate outcomes when there is demand uncertainty, so that every bid is payoff-relevant with positive probability. But first, we investigate which outcomes, if any, are stable under our price adjustment process.

### 4.1.3 Computation of the Stable Equilibrium

In order to further analyze the behavior of the dynamics specified by (1), we have conducted numerical simulations for a range of parameter values. This section reviews the results. To simplify the computation, we start the simulations with symmetric initial strategy distributions, allowing us to work with a single difference equation. We conjecture that the results would not change substantially with asymmetric initial distributions, but we have not yet checked this.

#### The Low Inertia Case

In this scenario, \( \alpha \) was set to 0.3, so each generator places a weight of 30% on approximately optimal responses to the most recent pattern of bidding by its opponent. Various values of \( \kappa \) were tested; the results presented are for \( \kappa = 100 \), so a bidding strategy becomes about 2.7 times less likely to be used for each decrease of 0.01 in its expected profits. (For this game, a 0.01 decrease in absolute profits is roughly equivalent to 1% of total profits.) The initial strategy distributions used were uniform over the set of feasible strategies (but the results do not appear to be very sensitive to the initial conditions). Average market-clearing prices along various other measures are displayed in Figure 1. (The transient initial dynamics
are omitted.) For these parameters, the market settles into a cycle with a length of about twenty periods. At the trough of this cycle, generators are bidding both units in at prices near 1.06 with high probability. However, at these prices, second unit profits are both rare and low, so there is little incentive for generators to bid their second units competitively. Second unit prices begin to drift away from marginal cost. But this means that generators can get away with boosting prices on their first units - as they realize this, first unit prices begin to drift upward as well, as does the market-clearing price. But as first unit prices begin to rise, it becomes more attractive to bring second unit prices back down in an attempt to supply the entire market. This effect reins in first unit prices as well, and so prices gradually drift back down to around 1.06. At the peak of the cycle, the average market-clearing price is about 1.3 - 30% above the “competitive” price level. Even at the trough of the cycle, average prices are bounded well away from marginal cost. The average market price over the entire cycle is 1.16.

The High Inertia Case In this scenario, $\alpha$ was set to 0.01, so generators update their strategies relatively sluggishly. This might reflect various factors, such as bureaucracy and costs associated with evaluating and changing bids, or an internal preference for price stability, or perhaps a desire to stay below the regulatory radar by avoiding dramatic bid swings. The other parameters were set as for the previous case. Results of these simulations are presented in Figure 2. One can observe that the cycles persist, although their period is naturally much longer. The magnitude of the price swings has also diminished, particularly on the top end - the highest average price observed over the course of the cycle is now only about 16% above the competitive level. However the average market price over the cycle is approximately 1.12, lower than in the low inertia case, but not substantially so.

Low Inertia with Imperfect Profit Maximization In the two cases presented above, the generators were assumed to be capable of quite precise profit maximization. Because optimal responses generally involve undercutting, this tends to destabilize the market and encourage price cycles. If generators make mistakes more frequently, then undercutting will be more difficult, and prices may be more stable. Here, we set $\kappa = 20$, so that a 0.02 drop in the expected payoff of a strategy now means it is about 1.49 times less likely to be used. We assume relatively rapid strategy updating of $\alpha = 0.3$. Results are presented in Figure 3. In contrast with the earlier figures, in this case we include the transient initial dynamics to provide a flavor for the speed of convergence. As anticipated, price fluctuations are now virtually negligible relative to the earlier cases. Furthermore, average prices are substantially higher than in either of the previous cases at roughly 1.27.

Reducing the Price Cap In this scenario, we explore the effect of reducing the price cap from 3 to 1.5. The other parameters are as in the low inertia case ($\alpha = 0.3, \kappa = 100$). Results are in Figure 4. Here the outcome is quite different: the market-clearing price converges to the competitive level of 1 (actually, just below it). It is not entirely obvious why the reduction in the price cap should have such a substantial effect, since the new cap would not appear to constrain market-clearing prices, which were always less than 1.5 under the old cap. However, the price cap does help to constrain second unit prices from drifting too high when they are payoff irrelevant. This in turn makes it less attractive to gamble by raising one’s first unit price - the chance of failing to sell is higher.
**Demand Uncertainty** In this simulation run, we relax the assumption that demand is known to be equal to two units with certainty. Demand is assumed to be uniformly distributed on the interval \([1,3]\). Generators are assumed to be capable of dispatching fractional amounts of each units capacity as necessary in order to clear demand. We set \(\alpha = 0.1\), but otherwise the parameters are as in the baseline cases. The results are presented in Figure 5, and in a slightly different format from the other figures. The lower graph shows the average net profit (not market-clearing price, in this case) of a generator over time. The upper graphs are contour plots in \(p_1 - p_2\) space that indicate the most frequently used strategies at selected points in time. At the trough of the price/profit cycle (near Period 60), second unit bids are clustered around 1.5 and first unit bids are fairly evenly distributed between 0 and 1.5. Observe that the first unit bids only come into play if demand falls between 1 and 2. At this point in the cycle, generators are relatively indifferent between setting a low first unit price in order to be sure to dispatch all of the first unit’s capacity - with the risk of earning a low price if the competing firm does likewise - and setting a high first unit price but dispatching less than the unit’s full capacity in expectation. Second unit bids are now directly payoff relevant whenever demand falls between 2 and 3. In these cases, raising or lowering one’s second unit bid involves a standard tradeoff between increasing the chance of dispatching that unit at a profit and reducing the price at which the first (inframarginal) unit is dispatched. After prices have fallen far enough, driving down profits on the second unit when it is dispatched, this tradeoff begins to favor supply reduction. Second unit prices shoot up to between 2 and 2.5 (Periods 80 and 10). This in turn briefly induces the generators to boost their first unit prices to around 2 before the gradual process of price decline begins again.

Clearly, because capacity constraints are sometimes binding under demand uncertainty, marginal cost pricing would not be expected in this game. However, because equilibria in games with capacity constraints are notoriously difficult to compute, it would not be clear a priori how large the average markup should be expected to be. For example, the largest payoff that a generator could earn by simply offering all of its capacity at the cap price of 3 is just 0.75 (with probability 0.5, demand is between 2 and 3 in which case the generator dispatches 0.5 units on average). In contrast, the profit earned by each generator when both offer their full supply at a price of 1 (although this is no longer an equilibrium) is higher, at 0.875. From this, one might surmise that even if \((p_1 = 1, p_2 = 1)\) is no longer an equilibrium, prices in equilibrium might be close to this level on average in spite of the presence of capacity constraints. However, in fact, market-clearing prices average around 1.42 over the course of the price cycle, substantially above marginal cost.

**4.1.4 Discussion**

Our analysis of bidding behavior under step function bidding is admittedly quite limited. We have restricted attention to a very simple, stylized example and have analyzed one particular price adjustment dynamic. Nonetheless, the results are suggestive of effects that would persist in more sophisticated models. In particular, we have demonstrated that when profit-maximization is a bit noisy, Bertrand-like equilibria with prices at or near marginal cost need not be stable - even when they are essentially unique in the set of pure strategy Nash equilibria. Furthermore, this is true even in the limit as profit-maximization becomes perfect. We believe this notion that firms compute only approximate best responses to be an intuitive and appealing
one. One could alternatively posit that firms sometimes experiment with alternative strategies. Because in practice it is often only the market-clearing price (and not competitors’ bids) that is observed, both optimization errors and experimentation seem to be plausible features of generator bids.

One might infer that the stable cycles we observe represent the time average of mixed strategy equilibria of the games analyzed. Strictly speaking, however, this cannot be true for the case in which demand is certain. Because the strategy space is compact, the support of any mixed strategy must be bounded above. In particular, if there were a symmetric mixed strategy equilibrium, there would be an upper bound \( \bar{p}_2 \) on the support of second unit bids. Any generator bidding this upper bound would earn zero profit on its second unit, as it would never be dispatched. However, our stable outcomes have the property that generators dispatch their second units at a profit with positive probability, even in the noiseless limit. But this means that any generator bidding \( \bar{p}_2 \) could do strictly better with some lower bid, contradicting its inclusion in the mixed strategy. (This argument is a bit loose, but could be formalized and extended to asymmetric equilibria.) Therefore, our outcomes cannot be viewed literally as mixed strategy equilibria, although in a sense they may represent “approximate” mixed strategy equilibria.

4.2 Approximating Supply Function Equilibria with Discrete Unit Bids

The fact that the temptation to undercut one’s rival can lead to instability and cycles is familiar from the theory of capacity constrained price competition. The preceding simulations help to illustrate how instability and cycles persist when we replace the capacity constraints with an increasing unit cost and relax the constraint that a firm must offer all of its units at the same price. However, because the model used is still quite stylized, its implications for real-world power markets are at best suggestive. In this section, we develop and test a simulation model that more closely approximates real markets. In this setting, we can examine the extent to which activity in the simulated market can be represented by a supply function equilibrium. To preview the results, we find that in some cases the time-average behavior in the market is fairly well described by an affine SFE, but this average generally masks substantial and persistent cyclicality in prices. Moreover, market outcomes can be sensitive to the rule that firms use to update their supply schedules from one day to the next. This indicates a need for caution – without knowing more about the rules of thumb that real firms actually employ, market predictions based on theory and simulations should be treated tentatively.

4.2.1 The Model

Our simulations will model competition in supply functions between two generators for a single day. (The framework could be extended without much trouble to handle larger numbers of firms.) Each generator controls a number of discrete units. We assume that generation costs are quadratic, in the sense that a generator’s total cost of providing \( q \) units of power is \( \frac{1}{2}c_iq^2 \), where \( c_i \) is a cost parameter for generator \( i \). We also assume that there is a price grid, so that each generator’s supply bid consists of the number of units it is willing to supply at each point on the price grid.

Supply and demand for the day are cleared in 24 hourly markets. Each generator submits a single supply function at the beginning of the day which is binding over all of the hourly market decisions.
markets. (The restriction to a single supply bid has been a feature of several power markets including those in the UK and Australia. In others, notably the CALPX and CAISO markets, generators could submit separate bids for each hour.) The demand curve for each hour is linear with variation in load over the course of the day captured by different intercepts for each hour. Both the slope of the demand curve and its deterministic variation from hour to hour are public knowledge. There is a uniform price for all units dispatched in each hour.

Unplanned outages constitute the only exogenous source of uncertainty in the model. Each unit has a small, independent probability (typically 10%) of being out of service for the day. Before market-clearing, each generator’s supply bid is amended to account for out of service units. For example, if a generator has two units out of service, one which was offered at a price of 10 and another which was offered at a price of 15, its supply schedule is reduced by two units at price 15 and above and by one unit for prices between 10 and 15.

**Updating of supply bids**

Loosely, each generator will aim to adjust its supply schedule toward a best response to the residual demand it faced yesterday. Below, we discuss more precisely how a generator computes an approximate best response and the speed of its adjustment.

First, note that in computing a generator’s best response, we will implicitly assume that it can observe the residual demand curve it faced yesterday. Since this amounts to an assumption that it can infer its rival’s bids, this is a fairly innocuous assumption for markets in which bids are public; however, if the only information available to a generator consists of its own bid and hourly prices and quantities, then this assumption may be quite strong. A generator begins by calculating the profit-maximizing price and quantity on each hourly residual demand curve. If these price-quantity pairs trace out an upward sloping supply curve, then this is the generator’s profit-maximizing response to yesterday’s residual demand. More generally, when the price and quantity grids are relatively coarse, this set of optimal points may include pairs such as \((p_1, q_1)\) and \((p_2, q_2)\) where \(p_2 > p_1\) but \(q_2 < q_1\). In these cases, it is impossible to construct an upward sloping supply function that optimizes pointwise against each hourly residual demand curve. We assume that the generator constructs an approximate best response function that attempts to hit all of the hourly optima but deviates where necessary to avoid bending backward.

In describing a generator’s updated strategy, we will distinguish between the supply function \(s^t_i\) that it would like to submit if it could offer fractions of a unit at different prices and its bid \(b^t_i\) which has a whole number of units at each price. A generator’s updated strategy is composed of a weighted average of its desired bid yesterday and its best response:

\[
\Delta s^t_i = s^t_i - s^{t-1}_i = \beta (BR_i(b^{t-1}_i) - s^{t-1}_i)
\]

\[
b^t_i = \text{round}(s^t_i)
\]

The parameter \(\beta\) can be thought of as reflecting the degree to which a generator thinks its rival’s most recent behavior, as opposed to its longer term behavior, is the best indicator of how it will bid today. Alternatively, \(\beta < 1\) can be taken as a stylization of the idea that generators do not update their bids on a daily basis. The need to distinguish between \(s^t_i\) and \(b^t_i\) arises when \(\beta\) is small and individual units are relatively large. In this case, an updating rule that
relies only on \( b_i \) can get stuck: the term \( \beta (BR_i(b_i^{t-1}) - b_i^{t-1}) \) may round to zero units even when \( BR_i(b_i^{t-1}) - b_i^{t-1} \) is consistently non-zero. Using \( s_i^t \) in the updating rule means that the generator will eventually adopt even small strategy changes if the gain to doing so is persistent.

### 4.2.2 Simulations

This section presents the results of preliminary simulations of the model. We should note that the parameters have been chosen for illustrative value and are not intended to correspond to any real market. In the benchmark scenario considered, the cost parameters of the two generators are \( c_1 = .2 \) and \( c_2 = .4 \) respectively, and we assume that neither generator faces a binding capacity constraint. Prices lie on a 100 point grid between 0 and 20, and the hourly demand curve \( D_h(p) \) is given by

\[
D_h(p) = 10 + z(h) - p
\]

for \( h \in \{1, 2, ..., 24\} \). Load is assumed to vary linearly over the course of the day: \( z(h) = 2h \). Figure 6 shows the maximum and minimum hourly demand, as well as the aggregate marginal cost curve for the industry.

In the sequel, we will often make reference to the affine supply function equilibrium of this model. One advantage of working with a quadratic cost specification is the existence of an easily calculated SFE in linear strategies which in this case is given (approximately) by \( q_1 = 1.56p, q_2 = 1.26p \). In addition to possessing the virtue of convenience, the affine SFE is of interest because Baldick and Hogan (2001) have demonstrated that it is the only stable equilibrium under a particular form of perturbation.

Figure 7 presents snapshots of the generators’ bid functions over time, starting from the affine SFE on Day 1. For reference, dotted lines show the affine SFE. While the supply schedules remain in the general neighborhood of the affine SFE, they do not converge to it. Instead, they cycle between phases with relatively more (Day 82) and less (Day 400) competitive pricing. Furthermore, if the market starts further from the affine SFE, these cycles tend to be more pronounced. Figure 8 shows the evolution of the market starting from a different set of initial conditions.² The variance of the average daily price (over the first 500 days) is 0.174, close to three times as great as when the market starts at equilibrium (0.064). Only a fraction (0.035) of the daily price variation can be attributed to the exogenous plant outages; the rest is endogenous. This suggests that any initial price volatility in the market tends to be preserved by the strategic adjustments of the generators. Volatility is also exacerbated when the generators are more responsive to recent market conditions. Figure 9 shows daily average prices when generators place a relatively high weight (\( \beta = 0.5 \)) on responding to the previous day’s residual demand. For comparison, the price series generated by affine SFE bidding (including outage shocks) is overlaid. Both the additional volatility and the cyclicality induced by the best response dynamics are pronounced.

The simulations allow us to investigate the impact of unit start-up costs and other non-convexities. As an example, suppose a “plant” consists of a group of five generation units. A generator incurs a fixed startup cost of 20 in each hour that it dispatches any of its first five units, \( 2 \times 20 \) in each hour that it dispatches units one through five, plus any of its second five

²Actually, these initial conditions are the result of running the system for a few days with \( \beta = .5 \), starting from the affine SFE.
units, and so on.\footnote{This is a very simplistic portrayal of start-up costs – a more complete model would account for the fact that the number of times a plant must be started depends on whether or not the hours during which it operates are contiguous.} Figure 10 shows the price series with start-up costs (for $\beta = .5$), with the no start-up cost series from Figure 9 for comparison. The generators recover some, but not all, of these costs through higher prices – average profits fall to 168 and 131 (versus 184 and 144 without start-up costs). And perhaps more surprisingly, price volatility is substantially dampened – the variance drops from 0.166 to 0.045.

5 Conclusion

It is commonplace in the burgeoning literature on competition in deregulated power markets to make relatively broad, and sometimes \textit{ad hoc}, simplifying assumptions about how firms will behave. Indeed, given the complexity of some of these markets, some such assumptions are essential if any headway is to be made. However, on occasion, simplification can be misleading; the tendency to focus on a taxonomy of competition that includes only Cournot and Bertrand (and increasingly on supply function equilibria as well) is a good example of this. Quite often, power markets operate under relatively idiosyncratic rules that can generate a rich assortment of different competitive outcomes. In order to develop a better understanding for the spectrum of these possible outcomes, basic game theoretic modeling of particular market rules can be quite useful. This paper develops such models for market rules resembling those in the California power markets, and analyzes them from several angles of attack. Our analysis is preliminary, and could be extended in a number of directions. On equilibrium existence, for example, further work is needed on the interlinkages between markets and the potential impact of non-convexities (like start-up costs). It would also be useful, although difficult, to develop more complete analytical characterizations of equilibrium bidding behavior to complement our simulation results. Such a characterization would be helpful in assessing the robustness of modeling tools like supply function equilibria.

6 Appendix

Deriving a Lower Bound on Average Prices: the N Firm, Asymmetric Case

Suppose that there are $N$ bidders with capacity constraints $K_1, K_2, K_3, ..., K_N$ and define the total industry capacity $K_{total} = K_1 + K_2 + ... + K_N$. The construction of the lower bound is very similar to the one in the main text. We define $K_{-i}$ to be total capacity without Bidder $i$: $K_{-i} = K_{total} - K_i$. Then if Bidder $i$ chooses to price at $p^c$, it has sales whenever $Q^c > K_{-i}$. In this event, the overflow is $Q^c - K_{-i}$, so its expected sales are $E[\min\{Q^c - K_{-i}, K_i\} | Q^c > K_{-i}]$. We define

$$\pi^c_i = p^c \Pr(Q^c > K_{-i}) E[\min\{Q^c - K_{-i}, K_i\} | Q^c > K_{-i}]$$

Bidder $i$’s profit from pricing at $p^c$ must be at least $\pi^c_i$.

Once again, in an equilibrium realization for which the market-clearing price is $p$, total industry profits cannot be greater than total capacity times $p$, or $pK_{total}$. Therefore, the total
expected profits in any equilibrium must be less than or equal to

$$E[p_{K_{total}}] = \mu K_{total}$$

Thus, if $$\pi_i$$ is Bidder $$i$$'s expected equilibrium profit we must have both

$$\pi_i + \pi_2 + ... + \pi_N \leq \mu K_{total}$$

and

$$\pi_i \geq \pi^{c}_i$$

for every bidder $$i$$. Together these imply

$$K_{total}\mu \geq \pi^{c}_{total}$$

where $$\pi^{c}_{total} = \pi^{c}_1 + \pi^{c}_2 + ... + \pi^{c}_N$$. Therefore, we have the following bound on average prices:

$$\mu \geq \pi^{c}_{total}/K_{total}$$

If there is any positive probability that any of the bidders ever faces overflow demand, then at least one $$\pi^{c}_i$$ is positive, which implies that $$\pi^{c}_{total}$$ is positive, and the bound is positive as well.
Figure 1: Low inertia

Figure 1:
Figure 2: High inertia

Figure 3: Noisy Profit Maximization
Figure 6:

Figure 7:
Figure 8:

Figure 9:
Figure 10:

Daily Prices (with startup costs)

- red: no startup costs
- green: startup costs