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### DESIGN OF A MECHANICAL RESONATOR TO BE COUPLED TO A THERMOACOUSTIC STIRLING-ENGINE

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#### ABSTRACT

This paper describes the design of a mechanical resonator for a thermoacoustic Stirling-engine. The engine was previously run with a quarter-wavelength acoustic resonator. The advantage of the mechanical resonator is that it is compact and would dissipate less acoustic power. The mechanical resonator consists of a twin piston-spring assembly moving in opposite phase to cancel vibrations. The system uses flexure springs to suspend the piston in a cylinder leaving a narrow gap between them. The narrow gap acts as a dynamic seal between the fronts and back sides of the piston. Simulation calculations show that the mechanical resonator dissipates 40 % less acoustic power than the acoustic one. This will lead to more useful acoustic power output from the thermoacoustic Stirling-engine. In addition, the size of the system is reduced considerably.

#### INTRODUCTION

Thermoacoustic systems require an acoustic resonator to sustain an acoustic wave. The resonator determines the operation frequency of the system and confines the pressurized working gas. Different type of acoustic resonators, mainly  $\lambda/2$ - or  $\lambda/4$ -resonators, have been used [1-6]. These acoustic resonators are long, heavy, and dissipate acoustic power by thermo-viscous and non-linear phe-

nomena. These losses result in a decrease of overall efficiency of thermoacoustic systems and less useful acoustic power output from the engine. Attempts have been made to use alternatives for acoustic resonators. Poese et al. used a hybrid acoustic-mechanical system in a thermoacoustic refrigerator used for ice cream sales [7]. H. Sugita et al. used a mechanical resonator for a Pulse-tube engine (work amplifier) [8]. Backhaus et al. used the resonance frequency of a linear motor for a thermoacoustic electric generator [9]. One of the problems encountered in operating these systems is that they do not start spontaneously and some extra means has to be used to start the system.

In this paper a preliminary study of the possibility to use a mechanical resonator for a thermoacoustic Stirling-engine is presented. The design of the mechanical resonator and its comparison with the acoustic one will be discussed.

The remaining of this paper is organized as follows: Section 2 is devoted to the operation characteristics of the mechanical resonator. In section 3, the design and optimization procedure of the mechanical resonator is given. In the last section some conclusions are drawn.

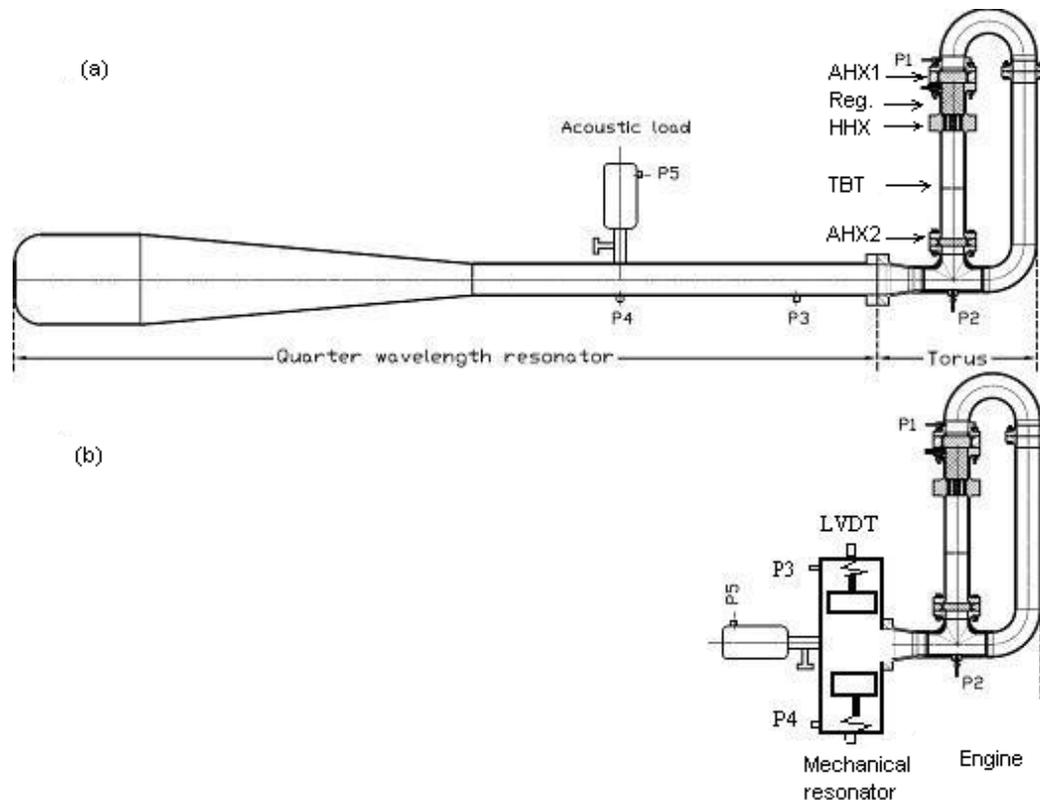


Figure 1. Acoustic resonator (a) and mechanical resonator (b) attached to the thermoacoustic engine. The “P”s are pressure sensors and LVDT is a displacement sensor.

## CHARACTERISTICS OF THE MECHANICAL RESONATOR

The mechanical resonator will replace an existing acoustic resonator attached to an existing thermoacoustic Stirling-engine. Experiments with the engine showed that most of the acoustic power produced by the engine is dissipated in the acoustic resonator and a smaller fraction is dissipated in a variable acoustic load (useful power). The engine uses the thermal power produced by electric cartridge heaters to generate acoustic power. Helium gas at an average pressure of 40 bars is used as the working medium at the operation frequency of 120 Hz. The mechanical resonator will replace the acoustic resonator and has thus to operate at the same pressure and frequency and the impedance has to match the thermoacoustic engine. A schematic illustration of the thermoacoustic engine attached to the acoustic and mechanical resonator is shown in Fig.1.

The engine consists mainly of a torus-shaped section attached to a quarter-wavelength standing-wave resonator

via a T-junction. A detailed description of this type of thermoacoustic engine can be found elsewhere [9,10,11].

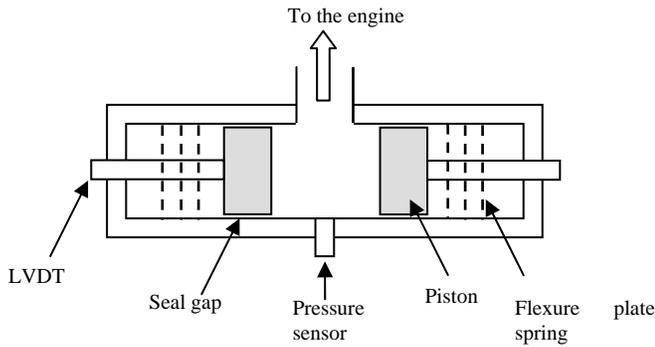
The quarter-wavelength resonator consists of three parts. The first part is made of a straight cylindrical tube with a length of 0.7 meter and a diameter of 5.5 cm (2” pipe), followed by a conical shaped part that increases the diameter to 16 cm over a length of 2.54 m. The last section is a stainless steel tube with a diameter of 16 cm and length of 43 cm, terminating in a cap. The large volume, formed by the last section, mimics an open end.

The design of the mechanical resonator incorporates the matching conditions at the T-junction where it is attached to the thermoacoustic engine. In the next section the design of the resonator will be discussed.

## DESIGN & OPTIMIZATION OF THE SYSTEM

The design of the mechanical resonator is inspired from the development of linear motors for cryocoolers. These systems have been made more reliable and maintenance free due to the use of linear flexures.

The mechanical resonator will be designed to interface with the engine. The impedance at the interface between the engine and the mechanical resonator should match correctly for proper operation. A schematic illustration of the system is shown in Fig.2. The mechanical resonator consists of a dual mass-spring system operating in anti-phase configuration. The oscillations of the pistons in dual opposed mode (anti-phase) in the same pressure vessel will cancel the vibrations and produce less noise. Each oscillator consists of a mass ( $m$ ) attached to a spring ( $K$ ) that can move in axial direction. The movement of the pistons will unavoidably generate dissipation as a consequence of frictional forces (mechanical, and viscous friction, and seal losses). This damping is generally expressed in terms of a mechanical resistance  $R_m$ .



**Figure 2. Schematic illustration of the mechanical resonator.**

There are four loss mechanisms which can occur in the mechanical resonator:

1. The internal mechanical friction and the flow through the springs cause dissipation of acoustic power.
2. There are also losses due to thermal relaxation in the compression space and back volumes.
3. T-branch at the interface to the thermoacoustic engine causes acoustic power losses due to minor losses.
4. The narrow annular gap between the piston and cylinder causes losses.

### Specifications

The operating parameters of the thermoacoustic engine with the mechanical resonator are derived from the model of the engine coupled to the acoustic resonator. Table 1 summarizes the operating parameters used in the design of the mechanical resonator.

**Table 1 Operation parameters of the torus engine coupled to the mechanical resonator.**

Average pressure (bar)	40
Frequency (Hz)	120
Drive ratio (%)	7
Working gas	Helium
Impedance at T-junction (Pa.s/m <sup>3</sup> )	4.5 10 <sup>6</sup>

### Resonance frequency

The resonance frequency of the mechanical resonator consisting of a mass  $m$  attached to a spring with a stiffness  $k$  is given by

$$f_o = \frac{1}{2\pi} \sqrt{\frac{K - \frac{|P_g| A \sin(\varphi - \theta)}{s}}{m}}$$

Here  $|P_g|$  is the amplitude and  $\varphi$  is the phase of the pressure difference across the piston,  $\theta$  is the phase of the volume flow in front of the piston,  $A$  is the area of the piston,  $s$  is the amplitude of the piston stroke,  $m$  is the moving mass, and  $K$  is the stiffness of the flexure springs.

The mass  $m$  consists of the entire moving mass including the piston, the shaft of the piston, and a part of the plates forming the spring. The stiffness  $K$  is the stiffness of the system. Flexure springs consisting of round discs made of spring steel are usually used in linear motors. They are reliable and they are used to center the piston inside the cylinder, providing a narrow radial gap between them. They have a very high stiffness in the radial direction and a controlled axial stiffness which forms the stiffness of the system. An example of a flexure spring is illustrated in Fig.3. In addition to the mechanical stiffness of the flexure springs, the back volume of the piston will also contribute to the stiffness of the system (gas-spring). The mass, flexure springs, and the back volume have to be designed to match the right resonance frequency.



Figure 3. Illustration of a flexure plate spring from reference [12].

### Impedance matching

The mechanical resonator has to match correctly the engine for proper operation. For the drive ratio specified in Table 1, the impedance is matched by guessing the piston area to get the right volume flow. This determines the diameter of the piston.

### DeltaEC model

The thermoacoustic computer code DeltaEC [13] is used to design the mechanical resonator taking into account the operation conditions given in Table 1 and the matching conditions at the T-junction. The DeltaEC model of the system consists of the thermoacoustic engine coupled to the twin mechanical resonator and an acoustic load, as shown in Fig.1b. Figure 4 shows the block diagram of the mechanical resonator as implemented in the DeltaEC model. A branch distributes the volume velocity between the variable acoustic load and the mechanical resonator. The variable acoustic load consists of a valve and a volume as shown in Fig.1. To model the twin mass spring system, the flow entering the mechanical resonator is split into two equal halves. The oscillating pressure amplitude at the intersection of the two masses is the same. The impedance seen by each of the piston face is equal to the ratio of the oscillating pressure and half of the oscillating flow into the mechanical resonator. The mass-spring system is modeled using an IESPEAKER segment where the electrical parameters are set to zero in order to model a passive mechanical oscillator. The width of the clearance gap is equal to 15  $\mu\text{m}$ . The mechanical mass and spring are varied in order to obtain a resonance frequency of about 120 Hz.

The trunk of the model continues further into the mechanical resonator. The flow entering into the mechanical resonator is split into two equal halves; one each flowing into the piston assembly using a BRANCH segment. The impedance value of the BRANCH segment is equal to the ratio of the pressure amplitude and the volume flow into each of the piston spring assembly. The pressure amplitude on each of the piston assembly would be identical.

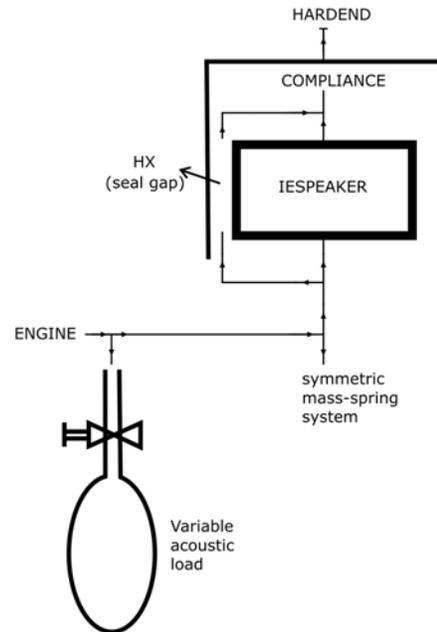
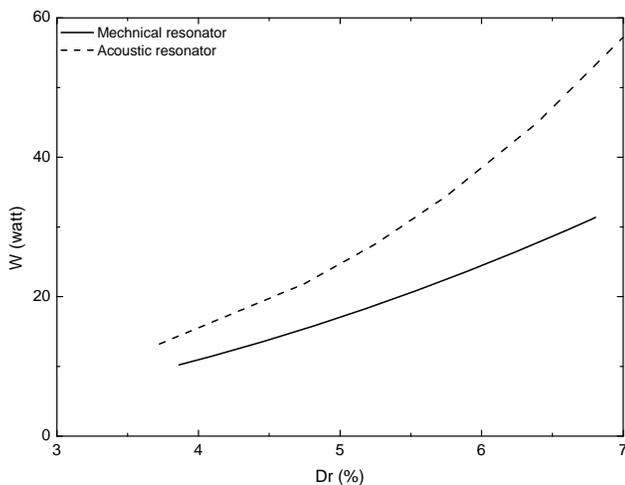


Figure 4. DeltaEC model of the mechanical resonator attached to the Stirling-engine.

The seal gap around the piston is implemented by a set of parallel plates (HX) and finally the bounce space behind the piston using COMPLIANCE. The DeltaEC flow sequence from the front face of the piston is as follows; A Tee splits the flow into two parallel paths. One path through the IESPEAKER simulates the piston and the spring assembly. The other path passes through a parallel plate HX with spacing equal to 15 micron and the length of path equal to 25.4 mm. The area of each parallel plate is taken equal to the outer surface area of the piston. Both these flows combine at the back of the piston and continue further into COMPLIANCE and shutting themselves up at the end cap with a HARDEND.

From the DeltaEC calculations, the moving mass required for a resonance frequency of 118.8 Hz is 0.45 kg and the mechanical spring stiffness  $K_f$  is  $9.610^4 \text{ N.m}^{-1}$  (six flexures are required to obtain this stiffness). The flexure spring used in the mechanical resonator is of oxford type and it has a mass of 30.4 g and a mechanical stiffness of  $1.6 \cdot 10^4 \text{ N.m}^{-1}$ . The moving mass of each linear flexure is estimated to be about one-third of the mass of the spring. The stroke amplitude is 4.18 mm at a drive ratio of about 6.8 %. The mechanical resistance due to air flow through the springs is assumed to be equal to  $2.0 \text{ N.s.m}^{-1}$ . For these conditions the volume behind the piston is about 1.065 liters. The diameter of the pistons is 0.055 m.

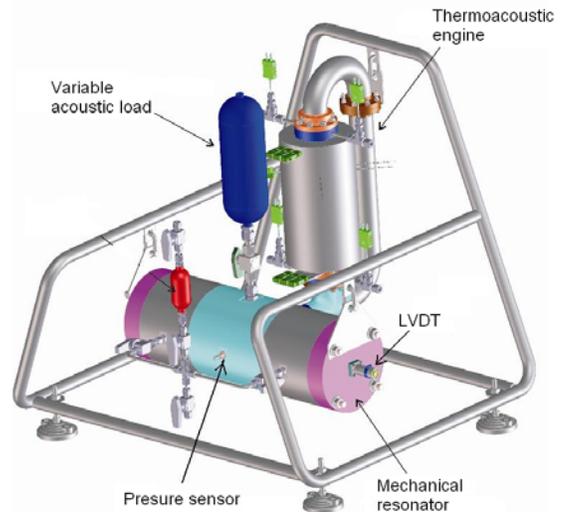


**Figure 5.** Calculation results of the acoustic power dissipated by mechanical resonator as function of the drive ratio.

Figure 5 shows the DeltaEC calculation results of the power dissipated in the acoustic and mechanical resonators as function of the drive ratio. The DeltaEC model of the acoustic resonator uses a relative roughness of  $10^{-4}$  and a minor loss factor for the cross-sectional changes due to the conical part of the acoustic resonator. For both resonators, the power dissipated increases with the drive ratio. The acoustic power dissipated in the acoustic resonator is higher than that dissipated in the mechanical resonator and that the difference increases with the drive ratio. The mechanical resonator dissipates 40 % less acoustic power than the acoustic one. This will lead to more useful acoustic power output from the thermoacoustic Stirling-engine. In addition, the size of the system is reduced considerably.

The characterization of the performance of the mechanical resonator requires knowledge of many quantities like displacement and pressures at different locations. The displacement of the piston will be measured using a Linear Variable Differential Transformer (LVDT) mounted on the pressure vessel. An aluminum rod connected to the moving mass of the resonator oscillates inside the coil of LVDT. Pressure sensors will be placed at different locations to measure the dynamic and static pressures. The next step is to build the system and tested by coupling it to the thermoacoustic Stirling-engine.

A CAD-illustration of the thermoacoustic engine attached to the mechanical resonator with the instrumentation is shown in Fig.6.



**Figure 6.** CAD-illustration of the thermoacoustic engine coupled to the mechanical resonator with instrumentation.

## CONCLUSIONS

A mechanical resonator is designed to be coupled to a thermoacoustic Stirling-engine, replacing an existing acoustic resonator. The mechanical resonator is compact and it consists of a twin piston-spring assembly moving in anti-phase to cancel vibrations. The system uses flexure springs to suspend the piston in a cylinder leaving a narrow gap between them. The gap acts as a dynamic seal between the fronts and back sides of the piston. The computer code DeltaEC is used to design and predict the behavior of the mechanical resonator. The acoustic power dissipated by the acoustic resonator is higher than that dissipated by the mechanical resonator and that the difference increases with the drive ratio. The mechanical resonator dissipates about 40 % less acoustic power than the acoustic one. In addition, the size of the system is reduced considerably

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## REFERENCES

- [1] G. W. Swift, "Analysis and performance of a large thermoacoustic engine", *J. Acoust. Soc. Am.* 92, 1515-1563 (1992).
- [2] T.J. Hofler, "Thermoacoustic refrigerator design and performance," Ph.D. dissertation, physics department, University of California at San Diego, 1986.
- [3] S. Garrett, J.A. Adef, and T.J. Hofler, "Thermoacoustic refrigerator for space applications," *J. of Thermophysics*

and Heat transfer, vol. 50, pp. 595, 1993. S. Backhaus, G.W. Swift, "A thermoacoustic Stirling heat engine: Detailed study", *J. Acoust. Soc. Am.*, 107, 3148-3166 (2000).

- [4] M.E.H. Tijani, J.C.H. Zeegers, and A.T.A.M. de Waele "Design of a thermoacoustic refrigerators". *Cryogenics* 42, 49-57 (2002).
- [5] M.E.H. Tijani, J.C.H. Zeegers, and A.T.A.M. Waele." Construction and performance of a thermoacoustic refrigerator" *Cryogenics*, Vol. 42, pp. 59-66, 2002.
- [6] T. Yazaki, A. Iwata, and T. Maekawa, "Traveling wave thermoacoustic heat engine in looped tube" *Phys. Rev. Lett.*, 81, 3128-3131 (1998).
- [7] M.E. Poese, R. Smith, and S. Garrett " Regenerator-base thermoacoustic refrigerator for ice cream storage applications," *J. Acoust. Soc. Am*, vol. 114, pp. 2328, 2003.
- [8] H. Sugita, Y.Matsubara, A.Kushino, T.Ohnishi, H. Kobayashi, and W. Dai "Experimental study on thermally actuated pressure generator for space cryocooler" *Cryogenics* 44, 431-437 (2004).
- [9] S. Backhaus, E. Tward, and M. Petach, " travelling-wave Thermoacoustic electric generator", *App. Phys. Lett.* 85, 1085-1087, (2004).
- [10] M.E.H. Tijani, S. Spoelstra, and G. Poignand "Study of a thermoacoustic-Stirling engine", *Acoustics* 08, (2008).
- [11] S. Backhaus and G. W. Swift, "A thermoacoustic Stirling heat engine: Detailed study", *J. Acoust. Soc. Am.* 10792, 3148-3166 (2000).
- [12] T. Trolhier, A. Ravex, and P. Crespi, "High Capacity Flexure Bearing Stirling Cryocooler On-Board the ISS", *International Cryocooler Conference* 12 (20021).
- [13] W. Ward, G.W. Swift, "Design Environment for Low Amplitude Thermoacoustic Engine", *J. Acoust. Soc. Am*, 95, 3671 (1994).